

Die Atmosphäre des AOIb-Überriesen η Leonis

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The Atmosphere of the A0Ib Supergiant η Leonis*

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We have measured the equivalent widths of 560 lines in the wavelength interval $3150 < \lambda < 6565 \text{ \AA}$, and with these measurements we have made a coarse analysis to get the initial parameters for the succeeding fine analysis. The observations are best fitted by a model atmosphere (radiative equilibrium, LTE, no line blanketing) with the parameters $\log g = 2.05 \pm 0.20$ and $T_e = 10400^\circ \pm 300^\circ \text{ K}$, and a microturbulence varying with the optical depth from 2 km/s at $\tau = 1.3$ to 10 km/s in the outermost layers. The relative temperature distribution in our final model is the same as that of a Mihalas model with the same parameters $\log g$ and T_e ; but Mihalas' method would give a value of T_e that is greater by 200° K .

The computed continuum agrees well with the photoelectric measurements of Schild and Oke. The computed color indices in the UBV system are $(U-B)_{\text{cal}} = -0.23$ [$(U-B)_{\text{obs}} = -0.25$] and $(B-V)_{\text{cal}} = -0.08$ [$(B-V)_{\text{obs}} = -0.04$]. The bolometric correction is $\text{B.C.} = 0.41$.

The profiles of the metallic lines of η Leo show a macrobroadening of about 15 km/s. We can obtain the same iron abundance from the Fe I and Fe II lines only if we use the new Fe I oscillator strengths of Garz and Kock.

The large helium abundance computed from the He lines that originate in the 2^3P and 2^1S terms is the result of an overpopulation of these levels because of departures from thermodynamic equilibrium; this can be explained by a chromospheric shell.

The average metal abundance is somewhat larger than the mean value for the Sun ($\log \epsilon - \log \epsilon_\odot \approx 0.25$). We have computed the luminosity as $L/L_\odot \approx 15000$, the radius as $R/R_\odot \approx 40$, and the mass as $M/M_\odot \approx 9$.

Key words: fine analysis — model atmospheres — abundance determination — η Leonis

I. Introduction

η Leonis is an A0Ib supergiant; no fine analysis has yet been made for an A0 star of this luminosity class, although there is a fine analysis for the even more extreme supergiant α Cyg (A2Ia) (Groth, 1961). It is particularly instructive to determine how much the details obtained for such an extreme supergiant atmosphere by the method of fine analysis — particularly the complicated velocity field — change when we go to a star with a less extended atmosphere but a not too different temperature.

We shall show that a consistent description of the equivalent widths of the strong and weak lines is possible with a model atmosphere having a depth-dependent microturbulence. The equivalent widths of the helium lines indicate departures from LTE and a temperature inversion in an outer shell of the atmosphere. The initial parameters necessary for the fine analysis (IV.) were obtained in a preliminary coarse analysis (III.) using our measurements (II.).

II. Measurements

1. Established Data and Observational Material

η Leo [$\alpha = 10^{\text{h}}04^{\text{m}}37^{\text{s}}$, $\delta = 17^\circ59'08''$ (1950)] is classified in the MKK system as an A0Ib supergiant; as a consequence of its position in the Hertzsprung-Russell diagram, it has an absolute magnitude of $M_v = -5.3$ (Allen, 1963). Its apparent visual magnitude is $m_v = 3.58$ (HD Catalogue), and the $UBVRI$ magnitudes (Arizona-Tonanzintla Catalogue) are:

V	$U-B$	$B-V$	$V-R$	$V-I$
3.53	-0.25	-0.04	0.12	0.13

The radial velocity is $v_R = +2.9 \text{ km/s}$ (Wilson Catalogue, 1953).

We have made microphotometer tracings of 13 spectra (Table 1) and measured the equivalent widths of 560 lines. Eleven spectra from the Mt. Wilson Observatory and two spectra from the Lick Observatory were graciously put at our disposal. The results of the measurements as well as information

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Table 1. *The spectrograms*

Plate number	Date	Dispersion (Å/mm)	Spectral region λ (Å)
Ce 2739	3.03.1942	10.0	3150–4576
Cd 2775 (a)	3.03.1942	2.8	3468–4029
Cd 2775 (b)	3.03.1942	2.8	4110–4670
Cd 2788 (a)	3.03.1942	2.8	3665–4270
Cd 2788 (b)	3.03.1942	2.8	4335–4880
Cd 7925 (a)	16.04.1952	2.8	3677–4179
Cd 7925 (b)	16.04.1952	2.8	4242–4740
Cd 7926 (a)	16.04.1952	2.8	3677–4164
Cd 7926 (b)	16.04.1952	2.8	4230–4740
Cd 3727 (a)	25.02.1945	2.8	3677–4080
Ce 3727 (b)	25.02.1945	5.8	5875–6375
Ec 2948	20.05.1964	10.0	5875–6565
Ec 2968	25.05.1964	10.0	5875–6565

concerning the methods of evaluation can be obtained from the Munich University Observatory upon request.

2. Mean Errors of Measurement and Comparison with Other Observations

The λ -dependence of the density curve has been investigated. The W_λ of the same lines, determined with different density curves in overlapping regions, rarely differ from each other by more than 5%. Table 2 gives the errors of the measured equivalent widths.

In the region $3850 < \lambda < 4850 \text{ \AA}$, our W_λ could be compared with the values of Vitrichenko and Kopylov (1962) (V and K). These authors evaluated five spectra of dispersion $23.4 \text{ \AA/mm}$ at H_γ . When $\Delta \log W_\lambda = \log W_{\lambda \text{ VK}} - \log W_{\lambda \text{ Wolf}}$ is considered as a function of $\log W_{\lambda \text{ Wolf}}$, there is only a weak dependence on W_λ . Likewise $\Delta \log W_\lambda$ does not show any clear dependence on λ . We have probably set the continuum systematically somewhat higher. Because of the substantially higher dispersion of our spectrograms, we used only our own measurements in the subsequent analysis.

Table 2. Mean error of the mean value of the equivalent width as a function of λ and W_λ

	$W_\lambda < 50 \text{ m\AA}$	$W_\lambda > 50 \text{ m\AA}$
$3150 < \lambda < 3680 \text{ \AA}$	only one good spectrum	
$3680 < \lambda < 4740 \text{ \AA}$	9%	5%
$5875 < \lambda < 6560 \text{ \AA}$	only two lines	9%

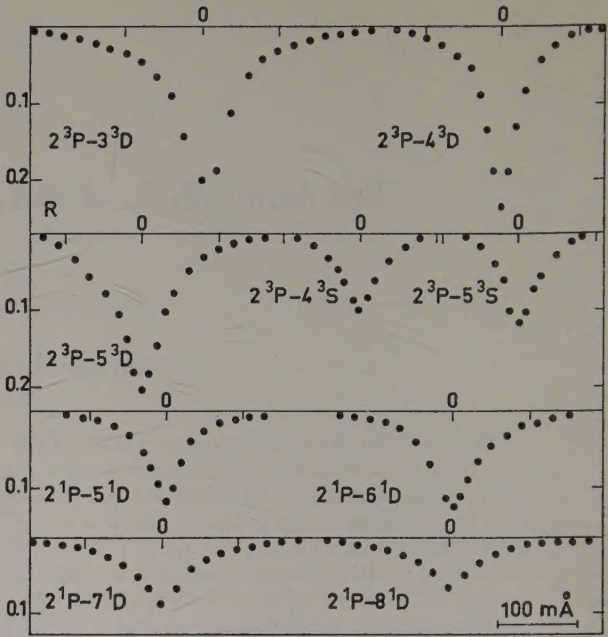


Fig. 1. Measured profiles of the He I lines

3. Summary of Further Measurements

The profiles of the Balmer lines $H_\alpha, H_\gamma, \dots, H_{10}$ were measured. H_α, H_γ and H_δ are presented in Fig. 6 (IV. 6a), together with the theoretical profiles. There is already an overlapping of the wings between H_{10} and H_{11} .

The Balmer jump was determined in a total of six spectra as $D = 0.47 \pm 0.03$. ($D = \log \frac{F_{>3650}}{F_{<3650}}$, where $F_{>3650}$ is the flux on the long-wavelength side of the Balmer jump and $F_{<3650}$ is the flux on the short-wavelength side). Chalonge and Divan (1962) give $D = 0.42$ for η Leo. We have adopted $D = 0.44$ from the absolute photoelectric measurements of the continuum (Oke and Schild, 1969).

In addition, profiles of the strongest metallic lines and of a few neutral helium lines were measured (Fig. 1). The central depths R_c of the strongest lines in the spectrum are plotted against λ in Fig. 2, together with the theoretical central residual intensities for Model 23 (see IV. 4). V and K give $R_c = 0.69$ for their wavelength region.

The existence of macroscopic motions in the atmosphere can be inferred from the profiles of the metallic lines. The coarse analysis (see III. 4) yielded a statistical Doppler velocity (thermal Doppler broadening and microturbulent broadening) of about



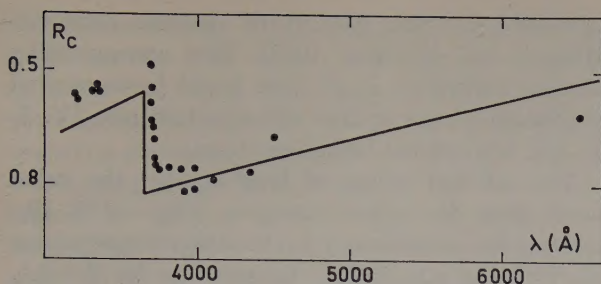


Fig. 2. Measured and theoretical central residual intensity

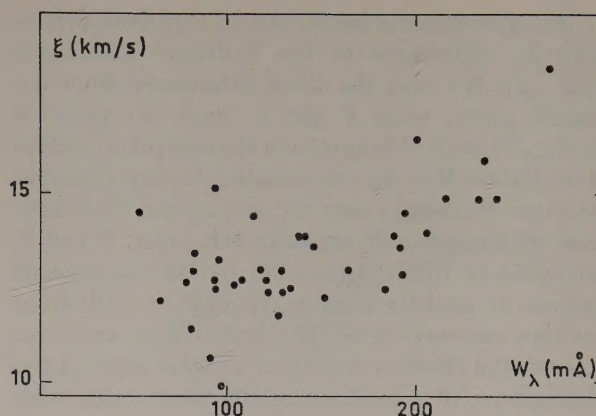
2.8 km/s. When we approximated the inner portion of the profiles of the metallic lines by a Gauss function (Groth, 1960) with $R \sim e^{-\left(\frac{\Delta\lambda}{\Delta\lambda_g}\right)^2}$, we encountered Doppler velocities $\xi = \left(\frac{\Delta\lambda_g}{\lambda}\right) c$ between 12 and 20 km/s (Fig. 3). Using the same procedure, the instrumental profile was determined from the weakest lines of the iron comparison spectrum to be no greater than 3 km/s. The measured value of ξ is thus essentially due to macroscopic motions in the atmosphere (see IV. 7).

For 34 lines the radial velocity v_R was also determined. The measurements were made with the Abbe comparator of the Munich University Observatory, using an electronic display of the line profile (Barwig, 1969). The average value is $v_R = 2.16 \pm 0.10$ km/s. The standard deviation of a single line is $\varepsilon = \pm 0.6$ km/s. At a dispersion of $2.8 \text{ \AA/mm}$, this means a setting error on the plate of about 0.003 mm . Errors of this order are to be expected in the photographic emulsion. If there are differential line displacements due to some kind of radial atmospheric motions, they are certainly small. For example, the mean value of the 9 Ti II lines that were used is 1.81 km/s , while that of the 20 Fe II lines is 2.43 km/s .

III. The Results of the Coarse Analysis

A coarse analysis was carried out using the usual methods (Unsöld, 1955) in order to determine the initial values of T_e , $\log g$, and the chemical composition, necessary for the fine analysis, and to obtain a general picture of the atmosphere. Our results can be compared with those of V and K.

The excitation temperatures T_{exc} (in the form $\theta_{\text{exc}} = 5040/T_{\text{exc}}$), the statistical Doppler velocities v_D (microturbulent velocity and thermal Doppler velocity), and the broadening parameters $\log \gamma/\Delta\omega_D$ (where γ is the damping constant and $\Delta\omega_D$ is the

Fig. 3. The Doppler velocities determined from the profiles as a function of W_λ

Doppler broadening) derived from the empirical curves of growth for the Fe I, Fe II, Ti II and Cr II lines are presented in Table 3, together with the corresponding θ_{exc} from V and K.

After eliminating the thermal Doppler velocity, we find $\xi_t = 2.25 \text{ km/s}$.

The empirical curves of growth for Fe I and Fe II were determined with the oscillator strengths of Corliss and Warner (1964) for Fe I, and with the values of Roder (1962), reduced according to Baschek, Kegel and Traving (1963), for Fe II (see IV. 6d). Otherwise, the same oscillator strengths were used in the coarse analysis as in the fine analysis. For the sources of the oscillator strengths, see IV. 6e.

Using the Inglis-Teller formula, modified according to Griem (Aller, 1963), we found $\log N_e = 11.87$. The following values were determined from the profiles of the Balmer lines:

	$\log N_e(\text{VK})$	$\log N_e(\text{Wolf})$	$\Delta \log N_e$
H γ	13.75	13.12	0.63
H δ	13.81	13.22	0.59

Table 3. Data obtained from the curves of growth

Curve	θ_{exc}	$\theta_{\text{exc}}(\text{VK})$	$v_D(\text{km/s})$	$\log \frac{\gamma}{\Delta\omega_D}$
Fe I	0.64	0.55	2.63	-1.0
Fe II	0.56	0.59	2.63	-0.7
Ti II	0.67	0.51	3.02	-0.7
Cr II	0.55	0.46	2.89	-1.1

The differences $\Delta \log N_e$ can be explained principally by differences in the hydrogen abundance $\log N_{0,2}H$. We used the value determined from the Balmer jump, while V and K used the value of $\log N_{0,2}H$ that is obtained from the equivalent widths of the Balmer lines by extrapolating to large principal quantum numbers, under the assumption that these lines are formed in an optically thin layer. V and K extrapolated from $H_\epsilon (n=7)$. In the subsequent analysis we used the mean value $\log N_e = 12.91$ from our own measurements. The Inglis-Teller value represents the electron density in a higher layer of the atmosphere; it is included in the mean value with half weight.

Using the Saha equation, we found an identical value of $\theta_{\text{ion}} = 0.54$ from the Fe II/Fe I and from the Mg II/Mg I ionization equilibria. The abundances of the elements obtained from the coarse analysis are presented in IV. 10 (Table 17). The large helium abundance is due to effects that are not considered in the coarse analysis. In our further calculations we therefore used the cosmic helium abundance. The gravitational acceleration was determined to be $\log g = 2.33$ in a purely spectroscopic fashion.

IV. Fine Analysis

1. Determination of the Model Parameters $\log g$ and θ_e

We constructed a grid of nine models (Table 4) within which the desired model should lie. The temperature variations $T(\bar{\tau})$ ($\bar{\tau}$ = optical depth) were

calculated by the method of discrete ordinates (Hunger and Traving, 1956). The corresponding pressure variations $P_g(\bar{\tau})$ were found by numerical integration of the hydrostatic equation (see IV. 3, Eq. (3), without the turbulence term).

The selected values of $\log g$ bracket the value found from the coarse analysis, $\log g = 2.33$. By equating the coarse-analysis excitation temperatures θ_{exc} with the equilibrium temperature for $\bar{\tau} = 0.1$, the values of θ_e were found from the remaining equations (Table 4). In this procedure, we used the value of $\theta_{\text{exc}} = 0.67$ from the Ti II curve of growth for Models 10 through 12, the mean value of $\theta_{\text{exc}} = 0.60$ from the Fe I, Fe II, Ti II and Cr II curves of growth for Models 13 through 15, and the value of θ_{ion} from the Mg I/Mg II and Fe I/Fe II ionization equilibria for Models 16 through 18. Since $\theta_{\text{exc}} = 0.67$ and $\theta_{\text{ion}} = 0.54$ are the extreme values, it was to be expected that the value of $T(\bar{\tau} = 0.1)$ in our final model should lie in this interval. For these nine basic models, we calculated the Balmer jump D , the W_λ of H_γ , H_δ and H_ϵ , ($U-B$), and the magnesium abundances $\log \epsilon_{\text{(Mg I)}}$ and $\log \epsilon_{\text{(Mg II)}}$ for the measured W_λ of the Mg I and Mg II lines, respectively (Table 5). By interpolation in the θ_e - $\log g$ diagram (Fig. 4), we determined the curves representing all values of θ_e and $\log g$ that correctly reproduce each observed quantity, or yield $\log \epsilon_{\text{(Mg I)}} = \log \epsilon_{\text{(Mg II)}}$.

For the calculation of the Balmer lines, see IV. 6a, and for the determination of the magnesium abundance, see IV. 6b. ($U-B$) and ($B-V$) were calculated

Table 4. The parameters of the nine basic models nrs. 10 through 18

Nr.	10	11	12	13	14	15	16	17	18
θ_e	0.564	0.562	0.562	0.498	0.495	0.494	0.448	0.445	0.444
$\log g$	1.5	2.0	2.5	1.5	2.0	2.5	1.5	2.0	2.5

Table 5. Data calculated from the basic models for the determination of θ_e and $\log g$

Model Nr.	D	$\log \epsilon_{\text{Mg I}}$	$\log \epsilon_{\text{Mg II}}$	$W_\lambda(H_\gamma)$ (mÅ)	$W_\lambda(H_\delta)$ (mÅ)	$W_\lambda(H_\epsilon)$ (mÅ)	$U-B$
10	0.605	6.90	8.00	5409	5271	5085	0.07
11	0.672	6.47	7.98	8622	8396	8377	0.14
12	0.678	6.21	8.01	12011	12007	11853	0.14
13	0.385	8.40	8.15	2910	2837	2737	-.23
14	0.482	7.83	8.05	5012	4910	4702	-.11
15	0.538	7.46	8.03	7502	7392	7190	-.06
16	0.255	9.85	8.50	2005	1951	1898	-.46
17	0.342	9.03	8.24	3727	3673	3556	-.35
18	0.394	8.53	8.14	5567	5539	5415	-.30
measured	0.44	—	—	4670	4869	4536	-.25

with the sensitivity curves of Mathews and Sandage (1963) for the *UBV* system and with the emergent radiative flux $F_0(\lambda)$. $F_0(\lambda)$ was determined by the procedure of Cayrel (1958). Between 3650 and 4000 Å, there is a strong depression of the continuum due to overlapping of the Balmer lines. In this wavelength region, we reduced the calculated $F_0(\lambda)$ by subtracting the fraction of the true continuum that was absorbed in the Balmer lines, as measured on the corresponding spectrograms (Table 1).

From the intersections of the curves in Fig. 4, we took $\theta_e = 0.477$ and $\log g = 2.15$ for a preliminary Model 20. The position of the final model is also indicated in Fig. 4, at $\theta_e = 0.485$ and $\log g = 2.05$.

2. Variation of the Microturbulence

For Model 20 we then determined abundances for some metallic lines, using the coarse-analysis value of the microturbulent velocity ($\xi_t = 2.25$ km/s) and pure radiation damping. (In IV. 6c, we shall show that only radiation damping is important for the metallic lines). In this procedure we used the Algol program for the quantitative analysis of stellar spectra by Baschek *et al.* (1966) (hereafter called the BHT program; cf. IV. 6).

When this calculation was carried out, the strong Si II lines led to an improbably high silicon abundance compared with the cosmic value, although the model already describes the remaining measured quantities, such as *D* and the W_λ of the Balmer lines, approximately correctly. Moreover, within a single multiplet there sometimes occurred systematic abundance differences of one to two powers of 10 between weak

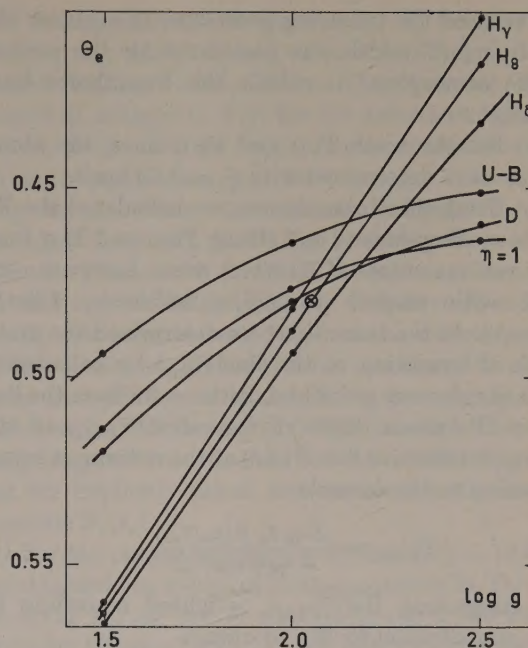


Fig. 4. θ_e - $\log g$ diagram for determination of the model parameters θ_e and $\log g$

and strong lines (Table 6), although we can assume that the relative *f*-values are correct. Furthermore, the Ti II, Fe II and Cr II lines on the short-wavelength side of the Balmer jump, which originate in a geometrically higher layer, gave systematically greater abundances than those on the long-wavelength side. The abundance differences $\Delta \log \epsilon$ are 0.23 for Ti II, 1.07 for Cr II and 0.45 for Fe II. The *f*-values of the lines with $\lambda < 3647$ are certainly not systematically smaller than the *f*-values of the lines with $\lambda > 3647$; nor can the chemical composition exhibit any variation with geometrical depth in the atmosphere. Such differences can, however, occur if an unanticipated "broadening mechanism" is operative in the geometrically higher layers. In fact, all the discrepancies we have described can be removed by assuming a depth-dependent microturbulence $\xi_t(\bar{\tau})$ that increases outwards (Groth, 1960). Since the shape of the curves of growth is influenced by this phenomenon as it would be by an additional damping, the large damping parameters $\log \gamma / \Delta \omega_D$ that are often observed in supergiants can also be explained. Examples are -1.0 to -1.5 for α Cyg (Groth, 1960), about -1.5 for φ Cas (Häfner, 1969), and about -1 for η Leo (see III. 4). We would expect to find pure radiation damping in these atmospheres, entailing values between -2.5 and -3 .

Table 6. Abundances of the elements calculated from weak and strong lines

Multiplet	λ (Å)	W_λ (mÅ)	$\log \epsilon$ (with $\xi_t = 2.25$)	$\log \epsilon$ (with $\xi_t(\bar{\tau})$ from Tables 7 and 8)
Cr II 3	3363.71	52.5	6.15	5.62
	3403.32	224.0	7.90	5.76
Cr II 30	4836.22	50.1	6.05	5.87
	4848.24	128.9	6.87	5.79
Fe II 37	4582.84	102.3	8.20	7.68
	4629.34	209.0	9.64	7.52
Ti II 20	4287.89	26.9	4.98	4.90
	4294.10	95.5	5.45	4.88
	4337.92	39.8	4.50	4.36
	4344.29	14.8	4.69	4.65

We used the following procedure to evaluate the function $\xi_t(\bar{\tau})$ which was postulated for the portion of the atmosphere in which the Fraunhofer lines originate:

a) For the weak Ti II and Fe II lines, the abundances were determined with $\xi_t = 2.25$ km/s.

b) Using these abundances, we calculated the W_λ of the medium-strong and strong Fe II and Ti II lines with various values of ξ_t , which were, however, constant with respect to $\bar{\tau}$ ($\xi_t = 2.25$ km/s, 6 km/s, 10 km/s). At the same time, we determined the mean depth of formation of the line ($\bar{\tau}_{\text{line}}$) by calculating for each reference point at a distance $\Delta\lambda$ from the line center the mean depth of formation $\bar{\tau}_{F(\Delta\lambda)}$ of the emergent radiative flux $F(\Delta\lambda)$ at the outermost layer, according to the formula

$$\bar{\tau}_{F(\Delta\lambda)} = \frac{\sum a_m \bar{\tau}_m B(x_m(\bar{\tau}_m))}{\sum a_m B(x_m(\bar{\tau}_m))}, \quad (1)$$

and integrating the $\bar{\tau}_{F(\Delta\lambda)}$, weighted according to their contribution to W_λ , to obtain

$$\bar{\tau}_{\text{line}} = \frac{\int \bar{\tau}_{F(\Delta\lambda)} R(\Delta\lambda) d\Delta\lambda}{\int R(\Delta\lambda) d\Delta\lambda}. \quad (2)$$

$R(\Delta\lambda)$ is the relative depth of the line at $\Delta\lambda$, $dx = (\kappa_\lambda + \kappa_L) dt$, κ_L is the line absorption coefficient, and κ_λ is the continuous absorption coefficient. The Cayrel points x_m (Cayrel, 1958) were used for the calculation of the Planck function B ; a_m are the corresponding weights. The BHT program was slightly extended for this purpose.

c) By interpolation, we then determined for each line the ξ_t for which the $W_{\lambda\text{cal}}$ agreed with the $W_{\lambda\text{obs}}$, and we associated this value of ξ_t with $\bar{\tau}_{\text{line}}$.

We obtained an appropriate description of all measured W_λ (that is, no dependence of the abundance on W_λ) with the $\xi_t(\bar{\tau})$ given in Tables 7 and 8.

3. Calculation of the Radiative Equilibrium Model

Using the definitive model parameters $\log g = 2.05$ and $T_e = 10\,386^\circ$, the corresponding temperature variation $T(\bar{\tau})$, and the microturbulence variation found in IV. 2, Model Nr. 21 was computed with Groth's programs, assuming a plane-parallel atmosphere in LTE and radiative equilibrium. The Planck function was used as a source function, and line blanketing was not taken into account. Thomson scattering enters only indirectly, since the sum of absorption and Thomson scattering appears in the tables of Bode (1965) in the term $\log\{(\nu_\nu + \sigma_\nu)\}$ ($\log P_\nu, \theta$). Rayleigh scattering plays no role in this temperature range. The procedure begins with the

integration of the hydrostatic equation, taking into account the radiation pressure $\sigma T_e^4/c$ and $\xi_t(\bar{\tau})$:

$$\frac{dP_g}{d\bar{\tau}} = \frac{g}{\bar{\kappa}_{\text{gr}}(\bar{\tau})} - \frac{\sigma T_e^4}{c} - \frac{d}{d\bar{\tau}} \left(P_g(\bar{\tau}) \frac{\xi^2(\bar{\tau}) \mu(\bar{\tau})}{2RT(\bar{\tau})} \right), \quad (3)$$

where $\bar{\kappa}_{\text{gr}}$ is the Rosseland opacity coefficient per gram, $d\bar{\tau} = \bar{\kappa} dt$, t is the geometrical depth, σ is the Stefan-Boltzmann radiation constant, c is the velocity of light, and $\mu(\bar{\tau})$ is the effective molecular weight. Then the monochromatic flux

$$F_\nu(\tau_\nu) = 2 \int_0^\infty B_\nu(\tau_\nu + t_\nu) E_2(t_\nu) dt_\nu - 2 \int_0^{\tau_\nu} B_\nu(\tau_\nu - t_\nu) E_2(t_\nu) dt_\nu \quad (4)$$

is calculated for every $\bar{\tau}$ and for 37 wavelengths λ_i between 911 Å and 40000 Å. An integration over frequency gives the integrated flux $F(\bar{\tau}) = \int F_\nu d\nu$.

The discrepancy between the calculated flux and the theoretical value

$$\Delta F(\bar{\tau}) = F(\bar{\tau}) - \frac{\sigma}{\pi} T_e^4 \quad (5)$$

is used to correct $T(\bar{\tau})$. In order to make $\Delta F(\bar{\tau})$ disappear, the correction obtained from the Edding-

Table 7. Radiative equilibrium model Nr. 21 for $T_e = 10386^\circ$ and $\log g = 2.05$, with the deviations of the radiative flux from constancy

$\bar{\tau}$	T	$\log P_e$	$\log P_s$	ξ_t	$\Delta F/F\%$
0.0000	8289	-2.186	-1.852	10.0	0.011
0.0005	8291	-1.199	-0.863	10.0	0.015
0.0010	8292	-0.913	-0.576	10.0	0.019
0.0030	8295	-0.482	-0.142	10.0	0.028
0.0050	8296	-0.292	0.054	9.9	0.030
0.0125	8306	0.028	0.384	9.8	0.014
0.0200	8318	0.188	0.552	9.5	-0.013
0.0350	8376	0.386	0.758	8.8	-0.043
0.0500	8473	0.521	0.894	8.0	-0.042
0.1000	8756	0.788	1.157	5.8	-0.035
0.1500	8986	0.937	1.304	4.0	-0.001
0.2500	9375	1.098	1.449	3.0	0.030
0.3500	9700	1.191	1.541	2.8	0.039
0.5000	10102	1.304	1.646	2.6	0.128
0.6500	10465	1.390	1.728	2.4	0.123
1.3000	11650	1.640	1.974	2.0	0.490
1.9500	12556	1.807	2.140	2.0	-0.822
3.9750	14474	2.125	2.445	2.0	0.036
6.0000	15834	2.316	2.628	2.0	-1.245
12.0000	18523	2.636	2.939	2.0	-0.017
18.0000	20323	2.828	3.129	2.0	-0.276
22.0000	21131	2.921	3.221	2.0	-1.183

ton approximation for the radiative transfer problem,

$$-\Delta B(\bar{\tau}) = \frac{1}{2} \Delta F(\bar{\tau} = 0) + \frac{3}{4} \int_0^{\bar{\tau}} \Delta F(\bar{\tau}) d\bar{\tau} - \frac{1}{4} \frac{dF(\bar{\tau})}{d\bar{\tau}}, \quad (6)$$

is applied to the Planck function for the integrated flux (Unsöld, 1955), and $T(\bar{\tau})$ is then modified accordingly. This procedure is repeated until the model atmosphere is sufficiently close to radiative equilibrium: that is, until $\Delta F(\bar{\tau})$ is less than 1% for all $\bar{\tau}$. In Model 21 these differences are in general substantially less than 1% for $\bar{\tau} \leq 1.3$. This is important, because a relatively small ΔF in the outer layers causes considerable changes in the temperature, while for large $\bar{\tau}$ changes of about 1% have practically no influence on T .

4. Residual Intensities and the Temperature Range in the Outer Layers. Comparison with a Mihalas Model

Using Model 21, we calculated the central residual intensity R_c of the strongest line in the spectrum (the K line of Ca II) as 70%. In order to obtain the measured R_c of 80%, a $T(\bar{\tau} = 0) = 7560^\circ$ would be necessary, as can be verified by the relation $R_c = 1 - \frac{B_\lambda(\bar{\tau} = 0)}{F_\lambda(\bar{\tau} = 0)}$.

Table 8. The definitive model 23 for η Leo, with the Parameters $T_e = 10386$ and $\log g = 2.05$, and the corresponding temperature variation according to Mihalas

$\bar{\tau}$	T	$\log P_e$	$\log P_g$	ξ_t	$T(\bar{\tau})$ Mihalas
0.0000	7560	-2.204	-1.868	10.0	7224
0.0005	7568	-1.227	-0.882	10.0	7584
0.0010	7576	-0.949	-0.596	10.0	7643
0.0030	7609	-0.532	-0.156	10.0	7791
0.0050	7642	-0.344	0.046	9.9	7836
0.0125	7769	-0.016	0.394	9.8	7930
0.0200	7901	0.161	0.573	9.5	8011
0.0350	8177	0.386	0.786	8.8	8149
0.0500	8473	0.548	0.924	8.0	8269
0.1000	8756	0.802	1.172	5.8	8591
0.1500	8986	0.946	1.314	4.0	8849
0.2500	9375	1.104	1.455	3.0	9224
0.3500	9700	1.195	1.545	2.8	9504
0.5000	10102	1.306	1.649	2.6	9965
0.6500	10465	1.392	1.729	2.4	10327
1.3000	11650	1.640	1.974	2.0	11505
1.9500	12556	1.807	2.140	2.0	12488
3.9750	14474	2.124	2.445	2.0	14648
6.0000	15834	2.318	2.627	2.0	15770
12.0000	18523	2.636	2.939	2.0	18194
18.0000	20323	2.828	3.129	2.0	20073
22.0000	21311	2.920	3.221	2.0	21048

We can see that the K line attains R_c , because W_λ , but not R_c , changes when the calcium abundance is varied within certain limits. Therefore we made empirical changes in $T(\bar{\tau})$ for the outermost layers — where, to be sure, the departures from local thermodynamic equilibrium may be very large — in such a way as to obtain the measured depths for the strongest lines (Fig. 2 (II. 3)). Probably none of the measured lines with $\lambda < 3647 \text{ \AA}$ reaches saturation. In this way we established our definitive Model 23, which now must be tested against the measurements of the continuum and of the line spectrum. The temperature variation of Model 23 can be roughly compared with a Mihalas model (1966), in which Thomson scattering in the continuum and Balmer-line blanketing are explicitly taken into account in the source function $S_\nu(\tau_\nu)$.

For the parameters $\log g = 2.05$ and $T_e = 10386^\circ$, we obtained the values of $T(\bar{\tau})_{\text{Mihalas}}$ given in Table 8. In doing so, we had to convert to our $\bar{\tau}$ the values given by Mihalas for τ_{5050} at the reference wavelength $\lambda = 5050 \text{ \AA}$. The agreement between the two temperature scales is quite good. We worked with Model 23 in the subsequent calculations because $T(\bar{\tau})_{\text{Mihalas}}$ can be obtained only through a rough interpolation in a grid of models calculated for a series of θ_e - $\log g$ pairs with pressure variations that do not take into account our depth-dependent turbulence function.

5. The Continuum

For $3270 \leq \lambda \leq 6050 \text{ \AA}$, the continuum can be compared with the absolute photoelectric measurements of Oke and Schild (1969) (Fig. 5). The measured values and our theoretical values are plotted against $1/\lambda$ in the system of Oke (1964), in the form $-2.5 \log F_\nu$ (erg/cm²s $\Delta\nu = 1$) + c ; c is found to be 14.

If we take into account the depression of the continuum due to the overlapping Balmer lines in the region $3646 \leq \lambda \leq 4000 \text{ \AA}$, Model 23 represents the measured continuum satisfactorily until somewhat greater discrepancies are encountered in the red spectral region from about $5500 \text{ \AA}$ on. The Balmer jump is correctly reproduced by Model 23. The continuum of Model 23 yields color indices ($U-B$) and ($B-V$) in good agreement with the measured values, if we take into account the influence of the Balmer lines between 3646 and $4000 \text{ \AA}$.

$$(U-B)_{\text{cal}} = -0.23, \quad (U-B)_{\text{obs}} = -0.25,$$

$$(B-V)_{\text{cal}} = -0.08, \quad (B-V)_{\text{obs}} = -0.04.$$

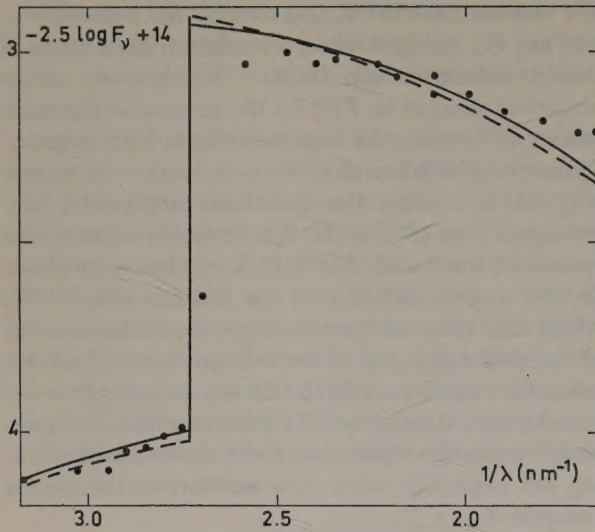


Fig. 5. Comparison of the continuum with the absolute photoelectric measurements of Schild and Oke

We calculated the bolometric correction to V , $B.C. = V - m_{bol}$, relative to the Sun as

$$B.C._{\eta \text{ Leo}} - B.C._{\odot} = 2.5 \log \frac{\pi F_{\eta \text{ Leo}} \int S_v(\lambda)_1 F(\lambda)_{\odot} d\lambda}{\pi F_{\odot} \int S_v(\lambda)_1 F(\lambda)_{\eta \text{ Leo}} d\lambda} = 0.34, \quad (7)$$

using the integrated fluxes $\pi F_{\eta \text{ Leo}} = 6.57 \times 10^{11}$ and $\pi F_{\odot} = 6.32 \times 10^{10}$ (erg cm $^{-2}$ s $^{-1}$), the $F(\lambda)_{\odot}$ obtained from Labs and Neckel (1967), and the sensitivity function $S_v(\lambda)_1$ (Mathews and Sandage, 1963). With $B.C._{\odot} = 0.07$, we find $B.C._{\eta \text{ Leo}} = 0.41$.

The range of $F(\lambda)$ in the V region agrees well with the photoelectric measurements (see Fig. 5). The principal uncertainty lies in the determination of $\pi F_{\eta \text{ Leo}}$, that is, of T_e . However, a Mihalas model which takes into account the Balmer-line blanketing would produce a difference of only about 200° in T_e , as can be seen from a comparison of the two temperature scales in Table 8.

6. The Line Spectrum and the Chemical Composition

a) The Balmer Lines.

The calculation of the Fraunhofer lines was carried out in accordance with the BHT program. The profile of the Balmer lines was calculated according to the asymptotic formula for the wings due to Griem (1962):

$$\phi(\Delta\lambda) = S(R, \Delta\lambda) = \frac{1 + [1 + (R/\Delta\lambda)^{-4}]^{-1/4}}{\Delta\lambda^{5/2}}; \quad (8)$$

$$R = A(n, m) \sqrt{\theta} [4.1007 - \log(\sqrt{P_e} n^2 \theta^{3/2})];$$

$$A(n, m) = 0.06479 (n^5 + m^5) / (n^2 m^2 \sqrt{n^2 - m^2})$$

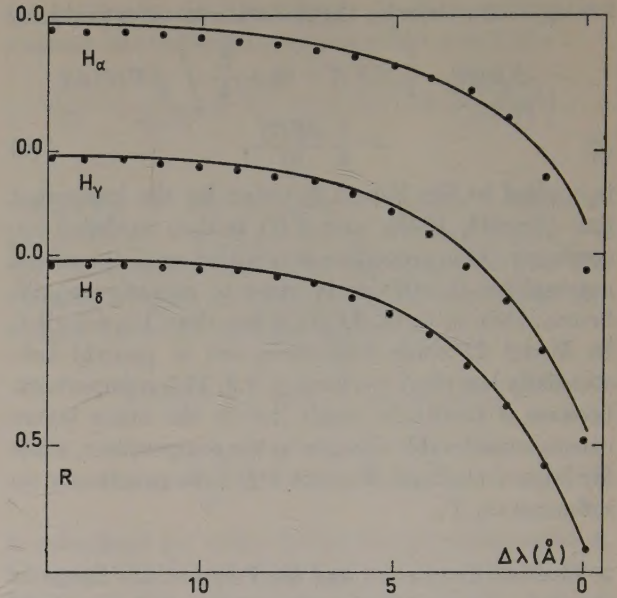


Fig. 6. Theoretical and measured profiles of H_{α} , H_{γ} , and H_{δ}

Table 9. Comparison of the observed equivalent widths of the Balmer lines with those calculated from model 23

Line	$W_{\lambda \text{ obs}}(\text{m}\text{\AA})$	$W_{\lambda \text{ cal}}(\text{m}\text{\AA})$	$\frac{W_{\lambda \text{ cal}} - W_{\lambda \text{ obs}}}{W_{\lambda \text{ cal}}} (\%)$
H_{α}	4319.5	4006.4	-7.7
H_{γ}	4670.5	4857.6	3.8
H_{δ}	4869.0	4796.6	-1.5
H_{ϵ}	4728.0	4947.2	4.4
H_{ζ}	4536.0	4697.5	3.4
H_{η}	4647.0	4671.5	0.5
H_{10}	4702.0	4652.2	-1.1

(where n is the principal quantum number of the upper level and m of the lower level). Only linear Stark broadening by ions and electrons was assumed. All other broadening effects, including the macroscopic motions observed in the metallic lines, have no influence on the wings of the broad Balmer lines. In practice, as close to the line center as $\Delta\lambda = 0.3 \text{ \AA}$, we encounter a pure linear Stark-broadening profile. Since we strove for the correct representation of the W_{λ} of H_{γ} , H_{δ} , and H_{ϵ} in our choice of θ_e and $\log g$, our model must yield essentially correct W_{λ} for the Balmer lines (Table 9). The comparatively large error of -7.7% in H_{α} is due to the poorer quality of the red spectrograms. No H_{α} emission can be established in $\eta \text{ Leo}$, as it can in $\alpha \text{ Cyg}$. The profiles of the Balmer lines calculated with Model 23 agree well with the measured profiles (Fig. 6).

b) The Line Absorption Coefficient κ_L of the Remaining Elements and Discussion of the Damping Constant.

The formula for the line absorption coefficients is:

$$\frac{\kappa_s}{\kappa_\lambda} = \frac{2\pi^{3/2} e^3}{m_e c} \frac{g_{r,s} f}{\Delta\omega_D} \frac{\varepsilon_i}{\Sigma\varepsilon_i} \varphi_n 10^{-\chi_{r,s}\theta} \frac{1 - e^{-\frac{h\nu}{\lambda T}}}{\kappa_\lambda} H(\alpha, v), \quad (9)$$

where e , m_e , c , and h are fundamental constants in the usual notation; $\frac{\varepsilon_i}{\Sigma\varepsilon_i}$ is the relative abundance of the element in question; $g_{r,s}$ is the statistical weight and $\chi_{r,s}$ the excitation potential of the lower level; κ_λ is the continuous absorption coefficient per nucleus; $(1 - e^{-h\nu/\lambda T})$ is the induced emission factor; and φ_n is the occupation number per quantum cell in the ground state of the n th ionization stage, as given by the ionization calculation. The broadening function is the Voigt function

$$H(\alpha, v) = \frac{\alpha}{\pi} \int_{-\infty}^{+\infty} \frac{e^{-y^2}}{\alpha^2 + (v-y)^2} dy,$$

with

$$\alpha = \frac{\gamma}{2\Delta\omega_D}; v = \left| \frac{\Delta\lambda}{\Delta\lambda_D} \right|; \Delta\lambda_D = \frac{\lambda v_D}{c}; v_D = \sqrt{\frac{2RT}{\mu_i}} + \xi_i^2;$$

where γ is the damping constant and μ_i is the atomic weight of the element in question.

γ is composed of radiation damping γ_{Rad} , electron damping γ_e (quadratic Stark broadening), and van der Waals damping γ_{vdW} . We took γ_{Rad} for the lines of He I, O I, Mg II, Al I, Si II and Ca II from Groth (1960), where it is calculated according to

$$\gamma_m = \Sigma \frac{A_{n,m}}{1 - e^{-h\nu/kT}} + \Sigma \frac{g'_n}{g_m} A_{m,n} (e^{h\nu/kT} - 1)^{-1} \quad (10)$$

for $T = 10000^\circ$. For the remaining elements, γ_{Rad} was calculated according to classical electron theory.

For γ_{vdW} , we have:

$$\begin{aligned} & \log \gamma_{\text{vdW}} (10^8 \text{ s}^{-1}) \\ &= 2/5 \log C_6 + 8.6735 + 7/10 \log \theta + \log P_H, \end{aligned}$$

where P_H is the partial pressure of the hydrogen atoms in the ground state and C_6 is the van der Waals interaction constant.

Using the approximation formula (Unsöld, 1955)

$$C_6 = 1.01 \times 10^{-32} \left[\left(\frac{13.6}{\chi - \chi_0} \right)^2 - \left(\frac{13.6}{\chi - \chi_b} \right)^2 \right], \quad (11)$$

where χ is the ionization potential, and χ_0 and χ_b are the excitation potentials of the upper and lower terms respectively, we were able to estimate the value

Table 10. Quantities used to estimate the importance of quadratic Stark broadening

$-\log C_4$	$\log \gamma_e$ ($\bar{\tau} = 0.1$) (10^8 s^{-1})	$\log \gamma_e$ ($\bar{\tau} = 1.0$)	λ (Å)	$\log \gamma_{\text{Rad}}$ (10^8 s^{-1})
10	1.4	2.3	3000	0.38
11	0.8	1.7	4000	0.14
12	0.0	1.0	5000	-0.05
13	-0.6	0.3	6000	-0.21
14	-1.2	-0.3	7000	-0.34
15	-1.9	-1.0		

Table 11. Silicon abundances with and without quadratic Stark broadening in the Si II lines

λ	W_λ	$\log \gamma_{\text{Rad}} - \log C_4$	Abundance with γ_e	Abundance for $\gamma_e = 0$
3853.66	186.1	0.08	13.47	8.16
3856.02	331.5	0.08	13.47	8.24
3862.60	269.4	0.08	13.47	8.04
6347.09	434.0	1.09	13.47	8.06
6371.36	330.0	1.09	13.47	7.91
4128.05	214.0	1.34	13.27	8.00
4130.88	204.2	1.34	13.27	7.77
5957.61	97.6	0.40	13.28	8.06
5978.97	124.2	0.40	13.28	8.02

Table 12. Helium abundances with and without quadratic Stark broadening in the He I lines

λ	W_λ	$\log \gamma_{\text{Rad}} - \log C_4$	Abundance with γ_e	Abundance for $\gamma_e = 0$
3964.73	56.4	0.15	10.84	11.26
4120.81	43.0	0.41	10.87	11.74
4387.93	63.0	1.30	9.60	11.09
4713.20	59.0	0.36	11.57	11.53

(The abundances are always referred to the hydrogen abundance $\log \varepsilon_H = 12$.)

of $\log C_6$ as -30 . Since hydrogen is very extensively ionized at all optical depths, γ_{vdW} is always several powers of ten smaller than γ_{Rad} .

For γ_e we have:

$$\log \gamma_e (10^8 \text{ s}^{-1}) = 2/3 \log C_4 + 7.7621 + 5/6 \log \theta + \log P_e,$$

where C_4 is the appropriate interaction constant. Estimating the physical parameters for $\bar{\tau} = 0.1$ and $\bar{\tau} = 1$, we have calculated γ_e for several values of $\log C_4$. For comparison, we have added the classical radiation damping in the region $3000 < \lambda < 7000$ Å (Table 10). For most elements, $-\log C_4$, in so far as it is known, has values of 13 and greater; this means

that for most lines, quadratic Stark broadening does not play a particularly important role. This can be shown for Model 23 by the strong silicon lines, whose C_4 values are known (Table 11). This implies that the metallic lines for which no C_4 values are known can be calculated taking into account radiation damping alone. Only in the case of such extreme quadratic Stark broadening as is found, for example, in the helium lines is it inadmissible to neglect γ_e (Table 12). In addition, we took γ_e into account in the case of O I, Mg I, Mg II, Al I, Si II and Ca II.

c) The Helium Lines.

The He I lines are rather strong in η Leo. Only a few lines are broadened by the quadratic Stark effect alone, and for them the Voigt function can be used as an approximate broadening function. We have measured the W_λ of some of these lines ($\lambda = 5875.6, 4713.2, 4387.9, 4120.8, \text{ and } 3964.7 \text{ \AA}$) (Table 13).

γ_e can be calculated (cf. Hardorp, 1966) by means of the formula

$$\gamma_e = 2w(1 + 1.36\alpha^{8/9}\sigma^{-1/3}); \sigma = 2w\rho_m/v, \quad (12)$$

where w is the half-width for electron broadening, α is the ion broadening parameter, ρ_m is the mean distance between the perturbing ions, and v is the mean velocity of the ions. Griem (1964) gives tables of w and α as functions of T . From γ_e we obtain the $-\log C_4$ values, needed for the BHT program, by using $\gamma_e = 11.37 C_4^{2/3} v_e^{1/3} N_e$, where v_e is the mean electron velocity. In this way we calculated the helium abundance $\log \epsilon_{\text{He}}$. The mean value, $\log \epsilon_{\text{He}} = 11.50$, is 2 to 2.5 times greater than the cosmic abundance (for an explanation of this effect see IV. 8).

For the remaining He I lines, the $\Delta\lambda$ -dependence of κ_L/κ_λ cannot be described by the Voigt function. For a theoretical treatment of these lines, one should refer to the field-dependent relative intensities of the components of the corresponding terms and their field-dependent displacement (Messerschmidt *et al.*, 1967) as numerically calculated and tabulated, for example, by Pfennig and Treffitz (1965).

Table 13. The dependence of $\log \epsilon_{\text{He}}$ on the initial state

λ	$\log \epsilon_{\text{He}}$	Initial state
5875.6	11.90	2^3P
4713.3	11.53	2^3P
4387.9	11.09	2^1P
4120.8	11.74	2^3P
3964.7	11.26	2^1S

d) The Iron Abundance.

As in the case of magnesium, lines of two different ionization stages of iron have been observed in η Leo. A correct model must give the same iron abundance for the Fe I lines and for the Fe II lines. Using the Fe I oscillator strengths of Corliss and Warner (1964), we obtained $\log \epsilon_{\text{Fe I}} = 7.45$; and using the f -values for the Fe II lines of Roder (1962), corrected according to Baschek *et al.* (1963), we found $\log \epsilon_{\text{Fe II}} = 6.82$. With the new f -values of Garz and Kock (1969) for Fe I, and those of Richter (1969, private communication before publication, Institut für Experimentalphysik der Universität Kiel) for Fe II, the abundances $\log \epsilon_{\text{Fe I}} = 7.79$ and $\log \epsilon_{\text{Fe II}} = 7.78$ were obtained with Model 23.

The fact that the discrepancy is smaller for the Fe I lines is due to the dependence of $\Delta \log gf = \log gf_{\text{GK}} - \log gf_{\text{CW}}$ on $\chi_{0,s}$; $\Delta \log gf$ decreases from about -0.1 for $\chi_{0,s} \approx 0$ to -0.7 for $\chi_{0,s} \approx 3 \text{ eV}$. For 39 of the 47 measured Fe I lines, $\chi_{0,s} \leq 1.61$. In this way we can also explain the good agreement of Groth's (1960) stellar Fe II oscillator strengths for α Cyg with the f -values of Richter ($\log gf_{\text{Richter}} = \log gf_{\text{Groth}} - 0.01$). Here too, Fe I lines with small excitation potentials were systematically used for the abundance determination. The abundances calculated with $\log gf_{\text{CW}}$ (Table 14) reflect the dependence of $\Delta \log gf$ on $\chi_{0,s}$ in the range $0 \leq \chi \leq 1.5 \text{ eV}$. To be sure, the eight lines with $\chi \approx 2.8 \text{ eV}$ disturb this picture somewhat. For the Fe II lines, there is no correlation at all between $\chi_{i,s}$ and $\log \epsilon_{\text{Fe II}}$.

The Fe II lines with $\lambda < 3647 \text{ \AA}$ yield about the same iron abundance as those on the long-wavelength side of the Balmer jump, which shows that we have made the right choice of $\xi_t(\bar{\tau})$ (see IV. 2). The same is true of the Ti II and Cr II lines.

	$\log \epsilon_{\text{Fe II}}$	$\log \epsilon_{\text{Ti II}}$	$\log \epsilon_{\text{Cr II}}$
$\lambda > 3647 \text{ \AA}$	7.80	4.73	5.66
$\lambda < 3647 \text{ \AA}$	7.76	4.74	5.56

Table 14. The iron abundance as a function of the excitation potential

$\chi_{0,2}(\text{eV})$	$\log \epsilon$ with $\log gf_{\text{GK}}$	$\log \epsilon$ with $\log gf_{\text{CW}}$	Number of lines
0.05	7.72	7.61	9
0.95	7.64	7.44	17
1.50	7.77	7.32	13
2.80	8.18	7.50	8

e) Summary of the Lines Used.

In Table 15 we list the calculated abundances for all lines, together with their atomic data. The sources of the oscillator strengths are as follows:

H I	Menzel, D.H., Pekeris, Ch.L. 1936, <i>M.N.R.S.</i> 96 , 77.
He I	Mult. Nr. 11, Weiss, A.W. (see Kohl, 1964, <i>Z. Astrophys.</i> 60)
	Mult. Nr. 12, 14, 18, 51, 58; Treffitz, E., Schlüter, A., Dettmar, K.H., Jürgens, K. 1957, <i>Z. Astrophys.</i> 44 , 1.
	Rest: Green, N.C., Johnson, N.C., Kolchin, E.A. 1966, <i>Ap. J.</i> 144 , 369.
C II	Biermann, L., Lübeck, K. 1948, <i>Z. Astrophys.</i> 25 , 113
O I	Jürgens, G. 1954, <i>Z. Phys.</i> 138 , 613.
Mg I	Treffitz, E. 1950, <i>Z. Astrophys.</i> 28 , 67.
Mg II	Mult. Nr. 4, Biermann, L., Lübeck, K. 1948, <i>Z. Astrophys.</i> 25 , 325.
	Mult. Nr. 5, 9, 10; Groth, H.G. 1961, <i>Z. Astrophys.</i> 51 , 231 (Bates and Damgaard).
	Mult. Nr. 27, hydrogen values used (Groth, H.G. 1961, <i>Z. Astrophys.</i> 51 , 231).
Al I	Biermann, L., Lübeck, K. 1948, <i>Z. Astrophys.</i> 25 , 325.
Si II	Hey, P. 1959, <i>Z. Phys.</i> 157 , 79.
Ca II	Mult. Nr. 1; Treffitz, E., Biermann, L. 1952, <i>Z. Astrophys.</i> 30 , 275.
	Mult. Nr. 4; Groth, H.G. 1961, <i>Z. Astrophys.</i> 51 , 231 (Bates and Damgaard)
Sc II	Corliss, C.H., Bozman, W.R. 1962, NBS Monograph 53.
Ti II	Warner, B. 1967, <i>Mem. R. astr. Soc.</i> 70 , 165; when not available, we used Tatum's values reduced to the system of Warner, Tatum, J.B. 1961, Com. U. London Obs., No. 41.
V II	Warner, B. 1967, <i>Mem. R. astr. Soc.</i> 70 , 165; or values of Corliss <i>et al.</i> , reduced to the scale of Warner.
Cr II	Corliss <i>et al.</i>
Ni II	Groth, H.G. 1961, <i>Z. Astrophys.</i> 51 , 231 (<i>f</i> -sum rule).
Mn II	Corliss <i>et al.</i>
Fe I	Garz, T., Kock, M. 1969, <i>Astr. Astrophys.</i> 2 , 274.
Fe II	Absolute values of Richter, J. (private communication, Institut für Experimentalphysik der Universität Kiel, 1969).
	Values of Groth, H.G. 1961, <i>Z. Astrophys.</i> 51 , 231, reduced to this scale.
Co II	Groth, H.G. 1961, <i>Z. Astrophys.</i> 51 , 231.
Ba II	Goldberg <i>et al.</i> 1960, <i>Ap. J. Suppl. Series V</i> , 45 (after Bates and Damgaard).
Na I	Biermann, L., Lübeck, K. 1948, <i>Z. Astrophys.</i> 25 , 325.

f) The Interstellar Sodium Lines.

The sodium lines D_1 and D_2 yield the value $\log \epsilon = 7.77$, which is very high compared with the Sun ($\log \epsilon = 6.30$). A comparison of these lines with Si II lines of similar central depth R_0 on the same plate, Ce 3727(b) (Fig. 7), shows that they are considerably narrower. We therefore conclude that the sodium lines are interstellar. We have measured $W_\lambda(D_2) = 262 \text{ mÅ}$ and $W_\lambda(D_1) = 196 \text{ mÅ}$. Merrill

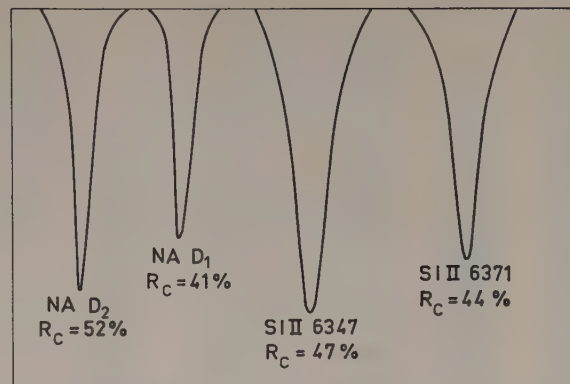


Fig. 7. Comparison of the interstellar sodium doublet with Si II lines of similar central depth R_0 on spectrogram Ce3727 (b)

et al. (1937), who also assume that this doublet is interstellar, give 260 and 160 mÅ, respectively, for these lines in the spectrum of η Leo. η Leo is about 600 pc away ($m - M = 8.83$), and has a galactic latitude of $b \approx 52^\circ$. Binnendijk (1952) has given a mean intensity-distance relation for interstellar calcium and sodium lines for various directions in the galaxy. The $W_{\lambda \text{ obs}}$ for the sodium lines in η Leo are what we would expect from this relationship. Whether interstellar absorption is also present in the H and K lines of Ca II cannot be established from the measurements of the profiles. Interstellar absorption in the K line should produce a W_λ of about 130 mÅ, as can be concluded from the observed intensity ratios of the interstellar sodium lines to the interstellar K line in stars which do not have the corresponding stellar lines. We have $W_\lambda = 603 \text{ mÅ}$ for the K line. Therefore the supplementary interstellar absorption cannot have any great influence. The K line yields a slightly greater $\log \epsilon_{\text{Ca}}$ than the Ca II 4 line at $\lambda = 3179.33 \text{ Å}$, which is measured with lower accuracy. The difference is $\Delta \log \epsilon = 0.15$.

7. The Profiles of the Fraunhofer Lines

The theoretical line profiles $R'(\Delta\lambda)$ do not agree with the measured profiles (Fig. 8). However, we have already pointed out in II. 4 that there are macroscopic motions with velocities ξ ranging from 12 to 20 km/s. In fact, we can give a good description of the measured profiles by convolving the calculated profiles $R'(\Delta\lambda)$ with a Gauss profile $1/\sqrt{\pi} \exp(-\Delta\lambda/\Delta\lambda_g)^2$ of Doppler width $\Delta\lambda_g = \xi\lambda/c$, where $\xi = 15 \text{ km/s}$ (Fig. 9). The evaluation of the con-

Table 15. *The abundances of the elements as calculated from the individual lines*

Mult. Nr.	λ (Å)	$\chi_{r,s}$	$\log gf$	W_λ	$\log \varepsilon$
He I					
5	3964.7	20.53	-1.30	43.0	11.26
11	5875.6	20.87	0.48	165.9	11.90
	5875.6		0.26		
	5876.0		-0.28		
12	4713.1	20.87	-1.03	59.0	11.53
	4713.3		-1.94		
16	4120.8	20.87	-1.81	43.0	11.74
	4121.0		-2.71		
51	4387.9	21.13	-0.89	63.0	11.09
C II					
6	4267.3	17.97	0.75	51.3	8.65
	4267.0		0.59		
O I					
3	3947.3	9.11	-1.26	42.7	8.08
	3947.5		-1.40		
	3947.6		-1.62		
5	4368.3	9.48	-1.72	34.7	9.05
10	6156.0	10.69	-0.79	284.0	9.36
	6156.8		-0.49		
	6158.1		-0.35		
Na I					
1	5889.9	0.00	0.13	262.3	7.81
	5895.9	0.00	-0.18	196.0	7.73
	interstellar!				
Mg I					
3	3838.3	2.70	0.51	129.0	7.36:
	3832.3		0.29	107.0	7.41:
	3829.4		-0.19	89.1	7.76
Mg II					
4	4481.3	8.83	0.76	502.0	7.70
	4481.1		0.48		
5	3848.2	8.83	-1.59	75.9	7.94
9	4434.0	9.96	-0.93	66.6	7.97
	4428.0		-1.23	42.7	7.76
10	4390.6	9.96	-0.56	107.0	7.87
	4384.6		-0.81	76.0	7.77
27	4331.9	11.58	-1.31	14.8:	8.05:
Al I					
1	3961.5	0.00	-0.28	58.9	6.35
	3944.0		-0.58	49.0	6.54
Si II					
1	3856.0	6.83	-0.74	331.5	8.24
	3862.6		-0.97	269.3	8.04
	3853.7		-1.69	186.1	8.16
2	6347.1	8.09	0.25	434.0	8.06
	6371.4		-0.05	330.0	7.91
3	4130.9	9.80	0.40	204.2	7.77
	4128.1		0.22	214.0	8.00
4	5978.9	10.03	-0.22	124.2	8.02
	5957.6		-0.52	97.6	8.06
Ca II					
1	3933.7	0.00	0.16	603.0	6.23
4	3179.3	3.14	0.52	209.0	6.08

Table 15 (continued)

Mult. Nr.	λ (Å)	$\chi_{r,s}$	$\log gf$	W_λ	$\log \varepsilon$
Sc II					
2	3613.8	0.02	0.48	39.8	2.96
	3630.7		0.32	15.5	2.64
3	3572.5	0.02	0.15	40.8	3.30
7	4246.8	0.31	0.19	49.0	2.91
14	4374.5	0.62	-0.06	26.9	3.02
Ti II					
1	3394.6	0.01	-0.65	263.1	6.30
	3387.8	0.03	-0.43	214.0	5.52
	3383.8	0.00	-0.08	195.0	4.79
	3380.7	0.05	-0.43	112.2	4.61
	3372.8	0.01	0.05	218.8	5.08
2	3254.3	0.05	-0.49	117.5	4.70
	3239.0	0.01	-0.04	186.1	4.81
	3236.6	0.03	0.14	158.5	4.39
	3234.5	0.05	0.26	229.2	4.99
	3217.1	0.03	-0.45	177.8	5.15
	3252.9	0.03	-0.49	155.0	4.99
	3242.0	0.00	-0.20	195.0	5.04
	3251.9	0.01	-0.51	158.5	5.03
6	3491.1	0.11	-0.95	64.6	4.75
	3489.7	0.13	-1.71	42.7	5.28
	3477.2	0.12	-0.80	75.9	4.72
	3461.5	0.13	-0.73	95.5	4.83
	3444.3	0.15	-0.61	72.5	4.51
	3500.3	0.12	-1.86	30.9	5.25
7	3346.7	0.13	-0.91	102.3	5.05
	3340.3	0.11	-0.42	89.2	4.43
	3335.2	0.12	-0.34	218.8	5.53
	3329.5	0.13	-0.26	173.9	5.00
	3322.9	0.15	-0.06	219.2	5.27
	3318.0	0.12	-0.98	60.3	4.72
9	3231.3	0.13	-1.31	95.5	5.38
10	3168.5	0.15	-0.13	158.5	4.73
11	4012.4	0.57	-1.68	87.1	5.33
	4025.2	0.60	-1.95	35.5	5.11
12	3813.4	0.60	-1.66	30.2	4.75
	3814.6	0.57	-1.54	47.9	4.86
13	3759.3	0.60	0.24	224.0	4.32
	3761.3	0.57	-0.01	75.9	3.59
14	3685.2	0.57	0.25	131.9	3.76
15	3573.7	0.57	-1.28	35.3	5.03
	3596.0	0.60	-0.84	44.7	4.73
	3587.1	0.60	-1.33	38.9	5.15
16	3372.2	0.60	-0.79	81.3	5.05
	3341.9	0.57	0.44	158.5	4.45
19	4395.0	1.08	-0.50	117.5	4.63
	4443.8	1.08	-0.74	91.2	4.67
	4450.5	1.08	-1.41	38.9	4.85
20	4287.9	1.08	-1.62	26.9	4.90
	4294.1	1.08	-0.90	95.5	4.88
	4337.9	1.08	-0.90	39.8	4.36
	4344.3	1.08	-1.67	14.8	4.65
24	3239.7	1.08	0.13	66.1	4.26
31	4468.5	1.13	-0.65	151.2	5.00
	4501.3	1.11	-0.79	93.4	4.76
34	3900.5	1.13	-0.13	166.0	4.64

Table 15 (continued)

Mult. Nr.	λ (Å)	$\chi_{r,s}$	$\log gf$	W_λ	$\log \varepsilon$
40	3913.5	1.11	-0.24	144.6	4.61
	3932.0	1.13	-1.37	43.7	4.96
	4417.7	1.16	-1.18	42.2	4.72
	4290.2	1.16	-0.79	81.4	4.71
41	4300.0	1.18	-0.46	128.9	4.72
	4307.9	1.16	-1.02	95.6	5.05
	4301.9	1.16	-1.11	53.7	4.79
	4312.9	1.18	-1.06	67.6	4.89
50	4315.0	1.16	-1.02	91.2	4.87
	4563.8	1.22	-1.86	91.2	4.87
	4399.8	1.23	-1.06	49.0	4.72
	3332.1	1.24	0.28	114.9	4.67
65	3321.7	1.23	0.11	85.1	4.58
	3315.3	1.22	-0.22	61.7	4.66
	3248.6	1.24	0.58	195.0	5.06
	3261.6	1.23	0.40	89.2	4.31
66	3272.1	1.22	0.07	53.7	4.28
	3271.7	1.24	0.14	95.5	4.64
	3278.3	1.23	0.16	102.3	4.67
	3282.3	1.22	1.22	100.0	4.75
72	3741.6	1.57	0.34	154.9	4.41
	3757.7	1.57	-0.04	87.1	4.32
	3776.1	1.57	-2.00	28.8	5.65
	3706.2	1.56	-0.29	135.0	4.91
73	4572.0	1.56	-0.34	123.0	4.77
	3218.3	1.56	0.42	66.1	4.28
	4028.3	1.88	-0.65	47.8	4.71
	4053.8	1.88	-0.80	52.5	4.92
88	3510.8	1.88	1.04	77.7	4.00
	3504.9	1.88	1.25	91.2	3.91
	3535.4	2.05	0.61	81.3	4.58
	4163.6	2.58	0.20	77.6	4.55
105	4171.9	2.59	0.13	57.6	4.44
	4174.1	2.59	-0.84	37.2	5.17
V II					
4	3589.7	1.07	-0.01	32.4	3.88
	3592.0	1.09	-0.01	39.8	3.99
	3556.8	1.07	0.11	52.5	4.01
	3267.7	1.07	0.52	87.1	3.91
7	3271.1	1.09	0.45	127.9	4.34
	3952.0	1.47	-0.52	35.5	4.05
	3916.4	1.42	-0.85	28.2	4.24
	3727.4	1.68	0.10	53.7	3.80
21	4005.7	1.81	-0.22	35.5	3.95
	4023.4	1.80	-0.35	28.2	3.95
	4035.6	1.79	-0.42	23.4	3.93
	4528.5	2.27	0.18	24.6	3.58
Cr II					
2	3511.8	2.47	-1.20	72.4	5.37
	3495.4	2.44	-1.22	138.1	5.94
	3433.3	2.42	-0.25	182.0	5.36
	3422.7	2.44	-0.10	251.5	5.97
3	3408.8	2.47	-0.05	245.7	5.86
	3403.3	2.42	-0.24	224.0	5.76
	3382.7	2.44	-0.43	154.9	5.29
	3363.7	2.42	-1.71	52.5	5.62
56	3391.4	2.41	-1.02	114.9	5.53

Table 15 (continued)

Mult. Nr.	λ (Å)	$\chi_{r,s}$	$\log gf$	W_λ	$\log \varepsilon$
4	3421.2	2.41	-0.29	234.2	5.93
	3368.1	2.47	0.15	89.2	4.16
	3358.5	2.44	-0.24	209.0	5.60
	3347.8	2.42	-0.72	151.3	5.53
9	3342.5	2.44	-0.32	173.9	5.34
	3339.8	2.42	-0.33	89.2	4.61
	3336.3	2.41	-0.72	177.9	5.76
	3328.4	2.41	-1.24	74.2	5.37
12	3180.7	2.53	-0.01	199.6	5.32
	3197.1	2.53	0.69	288.2	5.66
	3208.6	2.53	-0.16	67.7	4.28
	3209.2	2.53	-0.06	131.9	4.76
13	3214.4	2.53	-0.36	177.9	5.46
	3677.9	2.69	-0.85	131.9	4.95
	3631.7	2.69	-0.81	125.9	4.89
	3631.5		-0.45		
19	3713.0	2.69	-0.81	74.2	4.44
	3585.5	2.69	-0.65	158.5	4.87
	3585.3		-0.26		
	4052.0	3.09	-2.53	49.0	6.11
20	4076.9	3.09	-2.48	46.8	6.03
	4088.9	3.09	-3.43	30.2	6.75
	3738.4	3.09	-1.14	75.9	5.02
	3754.6	3.09	-1.44	69.2	5.26
21	3755.1	3.09	-2.76	63.1	6.52
	3765.6	3.09	-2.55	56.3	6.24
	3402.4	3.09	-0.83	104.8	5.68
	3393.9	3.09	-0.58	163.3	6.04
30	3379.8	3.09	-0.20	120.2	5.17
	3379.4	3.09	-0.58	114.9	5.52
	3378.3	3.09	-0.69	141.2	5.84
	3360.3	3.09	-0.03	173.9	5.48
31	3393.0	3.09	-0.82	75.6	5.59
	4824.1	3.85	-0.31	144.6	5.08
	4836.2	3.84	-1.89	50.1	5.87
	4848.2	3.85	-1.13	128.9	5.79
39	4876.4	3.84	-1.34	93.4	5.73
	4242.4	3.85	-1.02	117.5	5.64
	4261.9	3.85	-1.21	81.3	5.54
	4275.6	3.84	-1.33	69.1	5.54
44	4284.2	3.84	-1.64	56.3	5.71
	4252.6	3.84	-1.21	50.2	5.22
	4269.3	3.84	-2.06	32.4	5.83
	4565.8	4.02	-1.91	38.9	5.86
51	4555.0	4.05	-1.44	79.5	5.84
	4558.7	4.06	-0.31	186.1	5.48
	4588.2	4.05	-0.65	162.1	5.66
	4592.1	4.06	-1.37	83.2	5.81
63	4616.6	4.05	-1.51	72.5	5.85
	4618.8	4.06	-0.98	131.9	5.80
	4634.1	4.05	-1.19	114.9	5.88
	3295.4	4.16	-0.40	151.3	6.31
68	3234.1	4.28	-0.27	95.5	5.77
	3291.8	4.36	-0.70	95.5	6.25
	4012.5	5.64	-0.47	87.1	5.92
Mn II					
3	3497.5	1.80	-0.73	91.2	4.58

Table 15 (continued)

Mult. Nr.	λ (Å)	$\chi_{r,s}$	$\log gf$	W_λ	$\log \varepsilon$
	3496.8	1.80	-1.04	67.6	4.67
	3495.8	1.80	-0.65	130.8	4.85
	3488.7	1.80	-0.51	112.3	4.54
	3482.9	1.80	-0.32	158.6	4.75
	3474.0	1.80	-0.24	148.0	4.58
	3474.1				
	3442.0	1.80	0.05	282.0	5.75
blend					
Fe I					
4	3824.4	0.00	-1.26	32.4	7.76
	3859.9	0.00	-0.65	69.2	7.57
	3878.6	0.09	-1.30	50.5	8.08
	3886.3	0.05	-1.01	20.4	7.31
	3899.7	0.09	-1.35	21.4	7.69
	3922.9	0.05	-1.58	27.6	8.02
	3930.3	0.09	-1.40	77.6	8.44
5	3719.9	0.00	-0.48	47.9	7.19
	3745.6	0.09	-0.73	47.9	7.45
20	3820.4	0.86	0.24	112.1	7.55
	3825.9	0.91	0.06	79.5	7.49
	3840.4	0.99	0.00	43.7	7.63
	3865.5	1.01	-0.76	87.1	8.43
	3878.0	0.96	-0.69	50.2	7.98
21	3727.6	0.96	-0.44	32.4	7.51
	3743.4	0.99	-0.56	31.6	7.63
	3758.2	0.96	-0.17	70.8	7.34
	3763.8	0.99	-0.08	66.1	7.56
	3767.2	1.01	-0.21	33.9	7.32
	3787.9	1.01	-0.69	30.2	7.75
	3709.2	0.91	-0.44	25.7	7.47
22	3813.0	0.96	-0.79	19.7	7.60
	3850.8	0.99	-1.44	18.6	8.24
23	3881.2	0.86	0.41	50.2	6.83
	3618.8	0.99	0.07	42.7	7.87
24	3570.1	0.91	0.25	50.1	7.73
41	4383.5	1.48	-0.06	55.0	7.54
	4404.8	1.56	-0.14	42.7	7.65
	4415.1	1.61	-0.61	26.9	7.90
42	4271.8	1.48	-0.12	50.1	7.68
43	4005.2	1.56	-0.55	27.6	7.85
	4045.8	1.48	0.43	79.5	7.64
	4063.6	1.56	-0.04	63.1	7.79
	4071.7	1.61	-0.10	50.1	7.75
	4143.9	1.56	-0.55	67.6	8.35
45	3815.8	1.48	0.16	69.2	7.63
	3827.8	1.56	0.05	60.3	7.71
	3841.1	1.61	-0.05	44.7	7.65
	3903.9	1.56	-0.46	36.4	7.92
152	4187.0	2.45	-0.50	25.1	8.08
	4187.8	2.42	-0.54	38.0	8.49
	4235.9	2.42	-0.36	21.9	8.13
	4260.5	2.40	-0.04	37.2	8.07
	4198.3	2.40	-0.60	34.7	8.49
522	4199.1	3.03	0.12	25.2	7.96
608	3805.3	3.30	0.21	25.2	8.06
	3821.2	3.27	0.03	28.2	8.28

Table 15 (continued)

Mult. Nr.	λ (Å)	$\chi_{r,s}$	$\log gf$	W_λ	$\log \varepsilon$
Fe II					
1	3302.9	1.04	-3.61	100.0	7.69
	3303.5	1.09	-3.34	128.9	7.70
	3285.4	1.07	-3.93	120.2	8.19
4	3503.5	1.72	-3.95	42.7	7.88
	3508.2	1.72	-3.81	39.8	7.70
5	3425.6	1.66	-3.51	93.3	7.93
14	3746.6	2.27	-3.96	50.2	7.57
	3783.3	2.27	-3.45	138.9	7.86
16	3507.4	2.33	-3.36	39.8	7.63
	3416.0	2.27	-2.86	144.6	8.11
21	4119.5	2.53	-4.42	30.9	7.87
	4177.7	2.53	-3.73	97.7	7.93
22	4070.0	2.53	-4.58	33.2	8.08
	4124.8	2.53	-4.31	41.7	7.93
27	4128.7	2.57	-3.83	74.2	7.83
	4233.2	2.57	-2.58	298.2	7.89
	4351.8	2.69	-2.58	240.0	7.81
	4385.4	2.77	-2.67	66.1	6.69
	4416.8	2.77	-2.75	178.0	7.65
	4303.2	2.69	-2.69	199.5	7.69
	4273.3	2.69	-3.54	93.4	7.69
	4173.5	2.57	-2.74	195.0	7.56
28	4122.6	2.57	-3.52	97.7	7.75
	4178.9	2.57	-2.43	195.0	7.38
	4296.6	2.69	-3.14	141.1	7.75
	4258.2	2.69	-3.52	93.4	7.77
32	4278.1	2.68	-3.94	52.5	7.78
	4314.3	2.66	-3.33	87.1	7.50
	4413.6	2.66	-4.17	38.9	7.81
37	4472.9	2.83	-3.34	66.1	7.70
	4489.2	2.82	-3.79	129.0	8.38
	4491.4	2.84	-2.94	151.2	7.70
	4515.3	2.83	-2.63	190.7	7.57
	4520.2	2.79	-2.76	182.0	7.98
	4555.9	2.82	-2.60	204.0	7.67
	4582.8	2.83	-3.23	102.3	7.68
	4629.3	2.79	-2.14	209.0	7.62
	4666.8	2.82	-3.47	83.2	7.69
38	4508.3	2.84	-2.46	204.0	8.13
	4522.6	2.83	-2.49	234.2	7.76
	4541.5	2.84	-3.13	132.0	7.75
	4549.5	2.82	-2.30	363.3	8.37
	4576.3	2.83	-3.10	132.0	7.75
	4620.5	2.82	-3.47	89.1	7.74
	4583.8	2.79	-1.98	275.9	7.47
39	4138.4	2.82	-4.28	30.9	7.90
43	4731.4	2.88	-3.16	109.8	7.63
	4657.0	2.88	-3.79	61.7	7.83
44	4663.7	2.88	-3.99	33.9	7.67
46	5991.4	3.14	-3.72	91.2	8.19
74	6147.7	3.87	-2.34	120.1	7.47
	6247.6	3.87	-2.48	102.0	7.48
91	3436.1	3.95	-2.11	61.7	7.65
	3297.9	3.93	-2.07	64.6	7.61
112	3614.9	4.14	-2.07	55.0	7.68
114	3493.5	4.13	-0.98	141.1	7.39

Table 15 (continued)

Mult. Nr.	$\lambda(\text{\AA})$	$\chi_{r,s}$	$\log gf$	W_λ	$\log \varepsilon$
127	3468.7	4.14	-1.57	117.5	7.78
	3845.2	4.46	-2.47	89.1	7.75
	3865.0	4.48	-2.98	51.3	7.84
130	3781.5	4.48	-3.17	39.8	7.88
153	3827.1	4.71	-2.87	81.3	8.21
172	4044.0	5.55	-2.63	43.7	7.98
	4048.8	5.54	-2.17	69.2	7.84
173	3935.9	5.54	-1.61	109.8	7.72
186	4635.3	5.93	-1.67	102.2	7.90
	4625.9	5.92	-2.41	47.9	8.02
188	4111.9	5.93	-2.71	31.6	8.07
189	4061.8	5.93	-2.76	28.8	8.09
190	3938.9	5.89	-1.82	72.5	7.73
191	3975.0	5.93	-2.44	32.4	7.81
219	4598.5	7.77	-1.51	31.6	7.87
Co II					
2	3501.7	2.19	0.48	49.0	3.65
Ni II					
1	3513.9	2.85	-0.33	144.6	5.39
	3454.2	2.94	-0.56	70.8	5.00
4	3471.4	3.07	-0.93	69.1	5.43
	3576.8	3.06	-0.05	97.7	4.84
Ba II					
1	4554.0	0.00	-0.55	19.1	3.63

olution integral

$$\begin{aligned} R(\Delta\lambda) &= \int_{-\infty}^{+\infty} \frac{e^{-(\Delta\lambda/\Delta\lambda_g)^2}}{\sqrt{\pi}} R'(\Delta\lambda - \Delta\lambda') d\left(\frac{\Delta\lambda'}{\Delta\lambda_g}\right) \\ &= \frac{1}{\sqrt{\pi}} \int_{-\infty}^{+\infty} e^{-a^2} R(\Delta\lambda - \Delta\lambda_g a) da \end{aligned} \quad (13)$$

is performed by means of a seven-term summation (Kopal, 1961)

$$R(\Delta\lambda) \approx \frac{1}{\sqrt{\pi}} \sum_{j=i}^n H_j R'(\Delta\lambda - \Delta\lambda_g a_j), \quad (14)$$

where H_j and a_j are given by Kopal. The profile measurements are not adequate to decide the nature of the macroscopic motions. They can be explained by rotational broadening with an equatorial velocity of at least 20 km/s. According to Kraft (1960), rotational velocities of this order of magnitude are to be expected in Ib supergiants, if one assumes that the angular momentum in the *individual shells* of the star is conserved (for example, no significant loss due to friction), while the radius of the star changes in the course of its evolution. On the other hand, Huang and

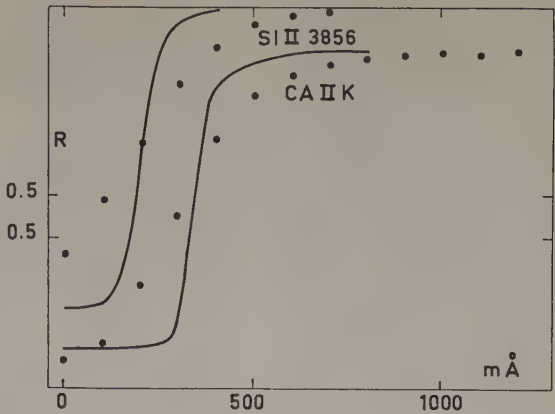


Fig. 8. Comparison of the measured profiles of the K line and the Si II line at 3856 Å with profiles calculated using Model 23 and the BHT program

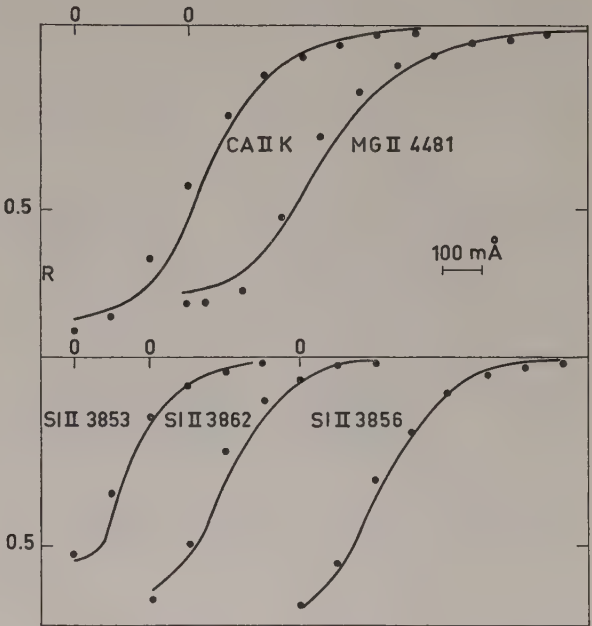


Fig. 9. Measured and theoretical profiles after inclusion of the macroscopic motion $\xi = 15$ km/s by means of convolution with a Gauss function

Struve (1960) see the cause of macroscopic broadening of the line profiles of supergiants in macroturbulent motions in the photosphere, with statistically distributed velocities.

8. The Chromospheric Shell

The helium lines yield the mean value $\log \varepsilon_{\text{He}} = 11.50$, which is too high with respect to the cosmic abun-

dance. The abundances derived from lines originating in the 2^1S and 2^3P states (cf. IV. 6c, Table 13) are systematically higher than that calculated from the 4387.9 Å line, which originates in the 2^1P state. This is to be expected as an effect of diluted radiation (as shown, for example, by the calculations of Wellmann, 1952), and indicates a chromospheric shell with a temperature sufficient for helium excitation. With appropriate values of the dilution factor W and the temperature, certain lines appear in absorption and others in emission (cf. Wellmann, 1952).

To get a specific idea, we chose a very simplified model with constant parameters: $W = 0.4$; electron temperature T_{el} = radiation temperature $T_r = 12000^\circ$ (thereby obtaining a high degree of excitation with a low degree of ionization of helium); $N_e = 10^{10}$ (corresponding to the density at the upper limit of the photosphere). Collisional excitation was not taken into account. The appropriate population factors b were interpolated from the tables of Roshdy (1962), who calculated them for the case of diluted Planck radiation ($b_{1^1S} \approx 5$, $b_{2^3P} \approx 5$, $b_{2^1S} \approx 6$, $b_{2^1P} \approx 1.5$). The extent of the chromosphere can be estimated as follows. The abundance $\log \epsilon_{He} = 11.09$ was calculated for the line λ 4387.9 Å. In this case, the influence of the chromosphere is smaller by a factor of 3 to 4 than in the case of the other lines, because of the small population factors of the 2^1P state, and it is not taken into account. With $\log \epsilon_{He} = 11.09$, we then calculated the W_λ of the remaining helium lines using Model 23, and we determined the differences $\Delta W_\lambda = W_{\lambda \text{ obs}} - W_{\lambda \text{ cal}}$ (Table 16), which according to our hypothesis are absorbed in an optically thin layer in the chromosphere.

In this way we first obtain the number of helium atoms in the 2^3P and 2^1S states, respectively, using:

$$\log NH_{0,s} = 17.053 + \log \Delta W_\lambda - 2 \log \lambda - \log f \quad (15)$$

where ΔW_λ is measured in mÅ and λ in Å. (The mean value for $s_1 = 2^3P$ is $\log NH_{0,s_1} = 12.90$ with moderate scattering; and for $s_2 = 2^1S$, $\log NH_{0,s_2} = 12.11$). Since the density of the helium atoms N_{He} can be determined from $\log \epsilon_{He} = 11.09$ in view of the fact that $N_H \approx N_e$ (N_H is the density of the hydrogen atoms, and hydrogen is extensively ionized in the

present case), H can finally be calculated by means of the equation:

$$\log NH_{0,s} = \log \left(N_{He} H \left(\frac{N_{HeI}}{N_{He}} \right) g \cdot \frac{b}{u} \right) - \chi_{0,s} \theta. \quad (16)$$

The ionization equilibrium calculation yields $\left(\frac{N_{HeI}}{N_{He}} \right)$; g is the statistical weight and u is the partition function, which in practice reduces to the population factor $b_{1^1S} \approx 5$. The height H of the chromosphere is computed to be about 5×10^6 km.

The assumption that the shell produces pure emission in H_α in an optically thin layer yields an upper limit for the total energy radiated in H_α :

$$E_{\text{tot}} = (A_{32} + B_{32} \pi F_\nu) h\nu N_3 H 4 \pi R_0^2 \\ = 8.5 \times 10^{32} \text{ (erg/s)}. \quad (17)$$

A_{32} and B_{32} are the transition probabilities for spontaneous and induced emission, respectively; πF_ν is the exciting radiative flux of the star; $R_0 = 40 R_\odot$ (IV. 9); and $N_3 = N_H b_3 g_3 \left(\frac{N_{H1}}{N_H} \right) 10^{-\chi_{30}}$, where $b_3 \approx 0.1$.

This result is to be compared with the total radiation of the photosphere in the region of H_α :

$$E_{\text{phot}} \approx W_\lambda 4 \pi R_0^2 \pi F_\lambda = 1.6 \times 10^{34} \text{ (erg/s)} \quad (18)$$

This would be the emitted energy if H_α did not exist in absorption.

If we take into account the fact that the central depth of H_α is about 60%, the energy radiated from the shell should give emission in H_α . Although emission in the hydrogen lines does not appear to be excluded, the observations at our disposal do not permit an unambiguous conclusion. For a more profound investigation of the excitation conditions and the structure of the chromosphere, it would no doubt be very productive to make use of the lines going from the metastable 2^3S state to n^3P ($n = 2, 3, 4$): λ 10830 Å (outside the measured spectral region), λ 3888.64 Å (buried in H_β), and λ 3187.74 Å (unidentifiable because of blends).

Because of the chromospheric effects, the He I lines arising in the 2^3P and 2^1S states are not suitable for the abundance determination, and we give the value calculated from the λ 4387.9 Å line, $\log \epsilon_{He} = 11.09$, in IV. 10 (Table 17).

9. Luminosity, Radius and Mass

With $M_v = -5.3$ and our calculated B.C. _{η Leo} = 0.41, we can compute $M_{\text{bol}} = M_v - \text{B.C.} = -5.71$

Table 16. The portion of the equivalent widths of the helium lines caused by chromospheric absorption

λ	5875.6	4713.3	4120.8	3964.7
ΔW_λ (mÅ)	66	21	28	9

and thus determine the luminosity $L_{\eta\text{Leo}}$ to be

$$\log L_{\eta\text{Leo}}/L_{\odot} = 4.17.$$

Using this value, and T_e for η Leo and the Sun, we find the radius of η Leo to be

$$\log R_{\eta\text{Leo}}/R_{\odot} = 1.58.$$

Using the gravitational acceleration and the radius, we determine the mass as

$$\log m_{\eta\text{Leo}}/m_{\odot} = 0.77.$$

The mass can also be estimated from the results of the theory of stellar evolution. If we move η Leo back along its evolutionary track (Hofmeister *et al.*, 1964) to the main sequence, taking into account the luminosity and temperature changes, we can use the mass-luminosity relation, for main-sequence stars, thus obtaining the somewhat higher value

$$\log m_{\eta\text{Leo}}/m_{\odot} = 0.95.$$

The difference can be explained by uncertainties in $\log g$ and M_v .

10. Summary of the Most Important Data

Temperature	$T_e = 10400^{\circ} \pm 300^{\circ};$ $\theta_e = 0.485 \pm 0.015$
Gravitational Acceleration	$\log g = 2.05 \pm 0.2$
Bolometric Correction	B.C. = 0.41
Electron Pressure	$\log P_e = 1.31$ at optical depth $\bar{\tau} = 0.5$
Gas Pressure	$\log P_g = 1.65$ at optical depth $\bar{\tau} = 0.5$
Microturbulence	from 2 to 10 km/s, depending on the optical depth
Macroscopic Motion	$\xi = 15$ km/s
Luminosity	$L/L_{\odot} \approx 15000$
Radius	$R/R_{\odot} \approx 40$
Mass	$m/m_{\odot} \approx 9$

11. Comparison of the Abundance Distribution of the Elements With the Sun and α Cyg

For the Sun we rely on the abundances of Goldberg *et al.* (1960), with corrections according to Zwaan (1962). The iron abundance has been recalculated using the new oscillator strengths of Garz and Kock (1969). A comparison of the helium abundance in η Leo with that in α Cyg is not very meaningful, on account of the difficulty of interpreting the helium

Table 17. *Abundances of the elements*

Element η Leo	η Leo (Coarse analysis)	Sun	α Cyg	Sirius
H	12.00	12.00	12.00	12.00
He	11.09	13.58:	—	11.63
C	8.65:	9.81:	8.72	8.19
O	8.83	8.51	8.96	9.36
Mg	7.80	7.65	7.36	7.81
Al	6.45	6.29	6.20	6.59
Si	8.03	7.88	7.45	7.88
Ca	6.16	5.96	6.15	6.47
Sc	2.96	—	2.82	3.19
Ti	4.73	4.99	4.68	5.13
V	3.97	3.82	3.70	3.88
Cr	5.61	5.14	5.36	5.67
Mn	4.65	3.73	4.90	5.57:
Fe	7.78	7.68 ^{a)}	7.60	7.62
Co	3.65:	—	4.64	3.73
Ni	5.18:	—	5.91	4.82
Ba	3.63	—	2.10	—
(Na)	7.77	—	6.30	—

^{a)} This value has been only roughly reduced to the new oscillator strengths.

lines as a consequence of the complicated broadening mechanisms and excitation conditions, which make their determination in both atmospheres somewhat problematical. The carbon and oxygen abundances agree well with the values for the Sun. The difference in oxygen abundance between η Leo and α Cyg can be partially explained by the fact that in α Cyg the infrared O I multiplets at λ 7771/6 and λ 8446 were also used, and they yield a systematically higher abundance. If we limit ourselves also in the case of α Cyg to the visual and photographic lines, we obtain $\log \epsilon_0 = 9.12$. In comparing the metals, we leave out the elements Mn, Co, Ni and Ba, which are uncertain both on account of the observations and on account of the f -values. On the whole the metal abundance in η Leo is somewhat higher than in the Sun and slightly lower than in α Cyg. On the average, we find $\Delta \log \epsilon \approx 0.25$ compared with the Sun, but $\Delta \log \epsilon \approx -0.1$ compared with α Cyg.

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